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# DOProC-based reliability assessment of steel structures exposed to fatigue<sup>☆</sup>

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## KEYWORDS

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Fatigue crack;  
Inspection of  
structure;  
Reliability  
assessment;  
Random variable;  
Probability of failure

**Summary** Attention to the fatigue cracks in steel structures and bridges has been paid for a long time. In spite of efforts to eliminate the creation and propagation of fatigue cracks throughout the designed service life, cracks are still revealed during inspections.

Fatigue crack damage depends on a number of stress range cycles. This is a time factor in the course of reliability for the entire designed service life. The failure rate increases in the course of time and the reliability decreased. If possible propagation of the fatigue crack is included into the failure rate, it is necessary to investigate into the fatigue crack and define the maximum acceptable degradation. Three sizes are important for the characteristics of the propagation of fatigue cracks. These are the initial size, detectable size and acceptable size. The theoretical model of fatigue crack progression is based on a linear fracture mechanics.

A tension flange has been chosen for applications of the theoretical solution suggested in the studies. Depending on location of an initial crack, the crack may propagate from the edge or from the surface. Regarding the frequency, weight and stress concentration, those locations rank among those with the major hazard of fatigue cracks appearing in the steel structures and bridges. When determining the required degree of reliability, it is possible to specify the time of the first inspection of the construction which will focus on the fatigue damage. Using a conditional probability, times for subsequent inspections can be determined.

For probabilistic calculation of fatigue crack progression was used the original and new probabilistic methods – the Direct Optimized Probabilistic Calculation (“DOProC”), which uses a

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purely numerical approach without any simulation techniques. This provides more accurate solutions to probabilistic tasks, and, in some cases, to considerably faster completion of computations. FCPProbCalc code has been developed using the aforementioned techniques. By means of FCPProbCalc (which stands for Fatigue Crack Probability Calculation), it is possible to carry out the probabilistic modelling of propagation of fatigue cracks in a user friendly environment and to propose a system of regular inspections which should reveal damage to the structure, using the described computational procedures.

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## Introduction

Probabilistic methods are used in engineering where a computational model contains random variables. Primary probability approaches are presented and developed for the modelling and analysis of uncertainty, and for evaluating the associated effects on safety and reliability (Kralik, 2010; Cajka and Krejsa, 2014a; Urban et al., 2014).

Probabilistic calculation model can be defined in probabilistic tasks in general as the function of random variables  $X_1, X_2, \dots, X_n$  expressed by statistical moments, parametric probability distributions or empirical probability distributions in form of non-parametrically defined bounded histograms (Cajka and Krejsa, 2014b). If the probabilistic calculation is focused into the structural reliability assessment (Kala, 2012), the structure safety must satisfy the condition of reliability, based e.g. on the assumption:

$$R \geq E \rightarrow Z = R - E \geq 0, \quad (1)$$

where  $R$  is resistance of structure,  $E$  load effect and  $Z$  safety margin. All of these variables is to be considered as a statistically dependent or independent random variables, representing uncertain quantities such as loads, material properties (e.g. Major and Major, 2009), geometric dimensions incl. manufacturing and assembly imperfections, and the environment properties in which designed structure performs its function.

Common notation of the theoretical time-invariant structural reliability problem - estimated failure probability  $p_f$ , can be defined relative to the criterion of reliability (1) as:

$$p_f = \int_{D_f} f(X_1, X_2, \dots, X_n) dX_1, dX_2, \dots, dX_n. \quad (2)$$

where  $D_f$  is failure area of the safety margin  $Z(X) < 0$  as a function  $f(X)$  of joint probability density of random variables  $X = X_1, X_2, \dots, X_n$  (Krejsa et al., 2013c). Determination of failure probability  $p_f$  based on the explicit calculation of the integral (2) is very complicated and generally unmanageable. For solution of the integral have been developed series of probabilistic methods (Krejsa and Kralik, 2015).

## Direct Optimized Probabilistic Calculation – DOProC method

The proposed method: Direct Optimized Probabilistic Calculation – DOProC, solves the integral (2) pure numerical way that is based on basis of probability theory and does not require any simulation technique. This is highly effective way of probabilistic calculation in terms of computation

time and accuracy of the solution for many probabilistic tasks. The novelty of the proposed method lies in an optimized numerical integration. In summary was published e.g. in Janas et al. (2009, 2010), for statistically dependent input random variables in Janas et al. (2015).

The algorithm of DOProC method has been implemented in several software codes, and has been used in many cases in probabilistic tasks and reliability assessments (Krejsa et al., 2016). For the application of the DOProC method can be used software titled ProbCalc (Krejsa et al., 2014a,b). In ProbCalc is relatively easy to implement analytical and numerical transformation probabilistic model of solved tasks. The ProbCalc is extensively useful in solving of probabilistic tasks of engineering practice, especially on probabilistic reliability assessment according to the current standards. The comprehensively methodology for probabilistic design and reliability assessment of anchor reinforcement in long mining and underground works was designed and utilized in program Anchor, with which is possible to realize the probabilistic calculation very flexibly (Krejsa et al., 2013d).

## Formulating the task of fatigue crack progression

Reliability of the bearing structure has been significantly influenced by degradation resulting (Lokaj and Klajmonova, 2014; Seitzl et al., 2011, 2012), in particular, from the fatigue of the basic materials. The fatigue crack that deteriorates a certain area of the structure components can be described with one dimension only –  $a$ .

In order to describe the propagation of the crack, the linear elastic fracture mechanics is typically applied (Anderson, 2005). This method uses Paris-Erdogan's law (Paris and Erdogan, 1963) and defines relation between propagation rate of the crack and range of the stress rate coefficient,  $\Delta K$ , in the face of the crack:

$$\frac{da}{dN} = C.(\Delta K)^m, \quad (3)$$

where  $C$ ,  $m$  are material constants,  $a$  is the crack size,  $N$  is the number of loading cycles and  $\Delta K$  range of the stress rate coefficient.

The fatigue crack will propagate in a stable way only if the initial crack  $a_0$  exists in the place where the stress is concentrated. This place is located at the edge or on the surface of the element.

The primary assumption is that the primary design should take into account the effects of the extreme loading and the

fatigue resistance (Vican et al., 2011) should be assessed then according (1). If such element is subject to the operating load, following cases can occur:

**safe service life** – the fatigue effects do not degrade the element by means of the fatigue crack,

**acceptable failure rate** – the fatigue effects degrade the element and decrease the load-bearing capacity of the element,

**acceptable failure rate** – fatigue effects are expressed as stress changes.

The calculation model of the fatigue crack propagation defines the stress when the maximum acceptable crack results in the constant resistance of the structure,  $R$ , that corresponds to the stress in the yield point  $f_y$ . The approach c) is more demonstrative and has been preferred to the approach b) because it describes the non-linear growth of the both stresses in the element under degradation.

When using (3), the condition for the acceptable crack length,  $a_{ac}$ , is:

$$N = \frac{1}{C} \int_{a_0}^{a_{ac}} \frac{da}{(\Delta K)^m} > N_{tot}, \quad (4)$$

where  $N$  is the number of cycles needed to increase the crack from the initiation size  $a_0$  to the acceptable crack size  $a_{ac}$ , and  $N_{tot}$  is the number of cycles throughout the service life.

The equation for the propagation of the crack size (3) needs to be modified for this purpose. The range of the stress rate coefficient,  $\Delta K$ , at the constant stress range,  $\Delta\sigma$ , is equal to:

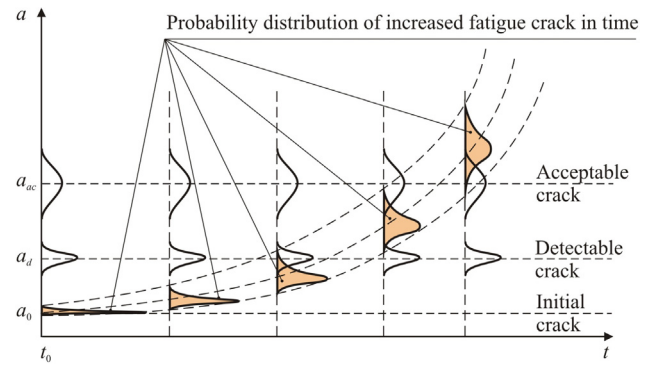
$$\Delta K = \Delta\sigma \cdot \sqrt{\pi \cdot a} \cdot F_{(a)}, \quad (5)$$

where  $F_{(a)}$  is the calibration function which represents propagation of the crack (for instance, from the edge or surface). After the change of the number of cycles from  $N_1$  to  $N_2$ , the crack will propagate from the length  $a_1$  to  $a_2$ . Having modified (3) and using (5), the following formula will be achieved:

$$\int_{a_1}^{a_2} \frac{da}{(\sqrt{\pi \cdot a} \cdot F_{(a)})^m} = \int_{N_1}^{N_2} C \cdot (\Delta\sigma)^m dN \quad (6)$$

If the length of the crack  $a_1$  equals to the initial length  $a_0$  (this is the assumed size of the initiation crack in the probabilistic approach) and if  $a_2$  equals to the final acceptable crack length  $a_{ac}$  (this is the acceptable crack size which replaces the critical crack size  $a_{cr}$  if the crack results in a brittle fracture - but in order to calculate the phenomenon  $a_2$  can be equal to the size of the detectable crack,  $a_d$ ), then the left-hand side of Eq. (6) can be regarded as the resistance of the structure –  $R$ :

$$R_{(a_{ac})} = \int_{a_0}^{a_{ac}} \frac{da}{(\sqrt{\pi \cdot a} \cdot F_{(a)})^m} \quad (7)$$



**Figure 1** Probabilistic growth of the fatigue crack in the course of time.

Similarly, it is possible to define the cumulated effect of loads,  $E$ , which is equal to the right side (random variable effects of the extreme load) (6):

$$E = \int_{N_0}^N C \cdot (\Delta\sigma)^m dN = C \cdot (\Delta\sigma)^m \cdot (N_0 - N), \quad (8)$$

where  $N$  is the total number of stress peak range,  $\Delta\sigma$ , when the crack size increases from  $a_0$  to  $a_{ac}$  and  $N_0$  represents the number of cycles in the time of the fatigue crack initiation (it is typically equal to zero).

It is possible to define a reliability function  $RF$  according (1):

$$RF_{(X)} = R_{(a_{ac})} - E_{(N)}, \quad (9)$$

where  $X$  is a vector of random physical properties such as mechanical properties, geometry of the structure, load effects and dimensions of the fatigue crack.

The analysis of the reliability function based on (2) gives a failure probability  $p_f$ :

$$P_f = P(RF_{(X)} < 0) = P(R_{(a_{ac})} - E_{(N)} < 0) \quad (10)$$

## System of inspection times

Because it is not certain in the probabilistic calculation whether the initial crack exists and what the initial crack size is and because other inaccuracies influence the modelling of the crack propagation, a specialized inspection is necessary to check the size of the detectable crack in a specific period of time (Liu et al., 2010; Chen et al., 2011; Kim and Frangopol, 2011; Soliman et al., 2013). The factor which influences most the time of inspection is the acceptable size of the crack (Krejsa and Tomica, 2011).

While the fatigue crack (see Fig. 1) is propagating, it is possible to define three random phenomena that are related to the growth of the fatigue crack and may occur in any time,  $t$ , during the service life of the structure. Then:

**$U_{(t)}$  phenomenon:** no fatigue crack failure has not been revealed within the  $t$  time and the fatigue crack size  $a_{(t)}$  has not reached the detectable crack size,  $a_d$ . This means:

$$a_{(t)} < a_d, \quad (11)$$

Probabilistic calculation of fatigue crack propagation in flange in tension of the cyclic loaded structures (Version 1.4.1.0)

Function Set up Help

Input data Results Inspections

Fatigue crack progression from: the edge

Number of years n starting / step / end values: 0 / 1 / 150

Design value of the limit probability pd: 2.277E-2

Width of the flange in tension bf [mm]: 400

Constant of material C: 2.2E-13

Thickness of the flange in tension tf [mm]: 25

Constant of material m: 3

Parameter epsilon for bounded parametric histogram: 1E-8

Number of intervals: 100

	Parametric / Raw data	Parametric distribution	Mu	Sigma	N int
Oscillation of stress peaks Delta S [MPa]	Parametric	Normal	30	3	100
Total number of oscillation of stress peaks per year	Parametric	Normal	1E6	1E5	100
Yield stress of material Fy [MPa]	Parametric	LogNormal_2P	280	28	100
Nominal stress in flange in tension Sigma [MPa]	Parametric	Normal	200	20	100
Initial size of the crack a0 [mm]	Parametric	LogNormal_2P	0.2	0.05	98
Detectable size of the crack ad [mm]	Parametric	Normal	10	0.6	100

Project:

RUN

10:48:52

Figure 2 FCProbCalc desktop with description of all input quantities which were entered into the system.

**$D_{(t)}$  phenomenon:** a fatigue crack failure has been revealed within the  $t$  time and the fatigue crack size  $a_{(t)}$  is still below the acceptable crack size  $a_{ac}$ . This means:

$$a_d \leq a_{(t)} < a_{ac}, \quad (12)$$

**$F_{(t)}$  phenomenon:** a failure has been revealed within the  $t$  time and the fatigue crack size  $a_{(t)}$  has reached the acceptable crack size  $a_{ac}$ . This means:

$$a_{(t)} \geq a_{ac} \quad (13)$$

Using the phenomena above, it is possible to define probability for their occurrence in any  $t$  time. Those three phenomena cover the complete spectrum of phenomena that might occur in the  $t$  time. This means:

$$P(U_{(t)}) + P(D_{(t)}) + P(F_{(t)}) = 1 \quad (14)$$

The probabilistic calculation is carried out in time steps where one step typically equals to one year of the service life of the construction. When the probability of failure  $P(F_{(t)})$  reaches the designed failure probability  $P_d$ , an inspection should be carried out in order to find out fatigue cracks, if any, in the construction element. The inspection in the  $t$  time may result in any of the three mentioned phenomena. The inspection provides information about conditions of the construction. Such conditions can be taken into account when carrying out further probabilistic calculations.

If no fatigue cracks are found, the analysis of inspection results gives conditional probability during occurrence. Using the inspection results for the  $t$  time, it is possible to define the probability of the mentioned phenomena in other times:  $T > t$ . For that purpose, the conditional probability should be taken into consideration. In order to determine the time for the next inspection, it is necessary to define the conditional probabilities  $P(F_{(T)} | U_{(t)})$  and  $P(F_{(T)} | D_{(t)})$  which can be expressed using the full probability law.

If re-distribution of stress from a point that is weakened by the crack is not taken into account, the crack propagation crack is usually rather high in the practical range of detectable values. If a fatigue crack is found during the inspection, it is necessary to monitor the safe growth of the crack or to take actions that will slow down or stop further propagation of the fatigue crack.

Those approaches are based on the using DOProC method for the calculation of probability of three basic phenomena, (11)–(13), for each year of operation of the construction. The details was published in Krejsa (2012a, 2013a) and the methodology was applied into the FCProbCalc code (Krejsa, 2012b). Using this software application, it is possible to monitor effectively and flexibly development of fatigue damage in steel structures, to determine times for inspections and to ensure that the construction will be fit for operation in terms of fatigue safety.

## DOProC probabilistic calculation

FCProbCalc has been developed using the aforementioned techniques. By means of FCProbCalc ("Fatigue Crack Probability Calculation"), it is possible to carry out the probabilistic modelling of propagation of fatigue cracks in a user friendly environment and to propose a system of regular inspections which should reveal damage to the structure.

The reference probabilistic calculation in FCProbCalc included the probabilistic assessment of a steel/reinforced concrete bridge from on the highway in a point where a longitudinal beam connects to a transversal beam (details see Krejsa, 2013b). The input quantities were determined deterministically or stochastically using non-parametric (empiric) and parametric probability distributions (see Fig. 2). The required reliability was described by the reliability index  $\beta = 2$  which corresponded to the designed probability of failure  $P_d = 0.02277$ .



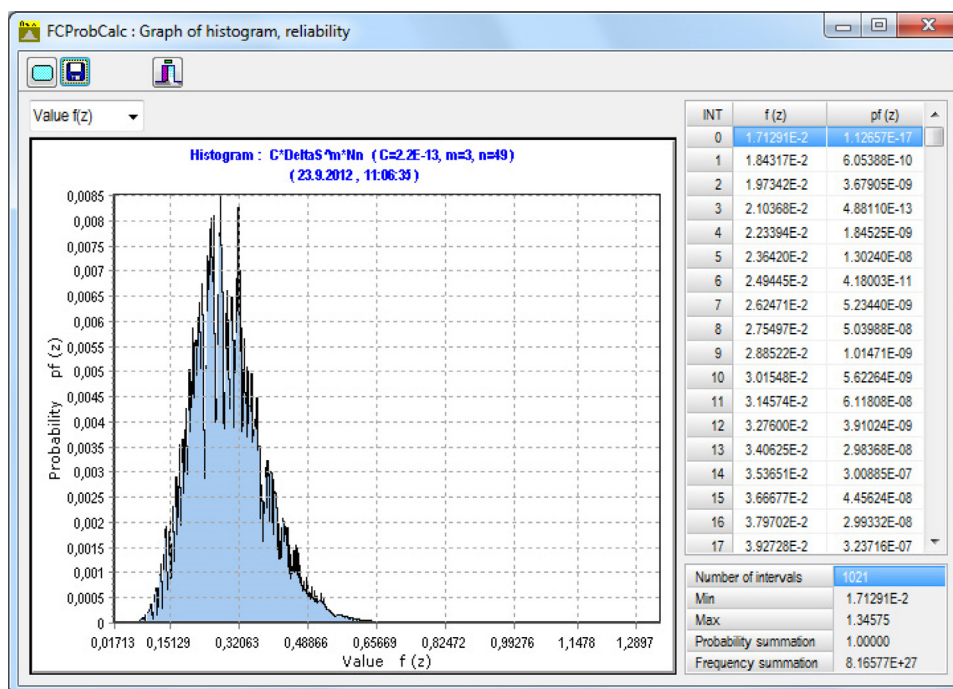


Figure 3 Resulting histogram for the S load effects for a bridge structure after 49 years of operation.

The probabilistic calculation was carried out for fatigue cracks propagating from the edge and surface. If a period of time is specified and the step is 1 year, it is possible to determine load effects,  $E$ , pursuant to (8) – see Fig. 3, resistance of the construction  $R_{(ad)}$  and  $R_{(aac)}$  pursuant to (7) – see Fig. 4 (so far, five types of numerical integration

are available) as well as the probability of elemental phenomena,  $U$ ,  $D$  and  $F$ , pursuant to (11) through (13) which are the basis for specification of inspection times.

Fig. 5 shows results of the probabilistic modelling of a fatigue crack from the edge. The curves describe dependence of the probability of failure,  $P_f$ , on time of operation

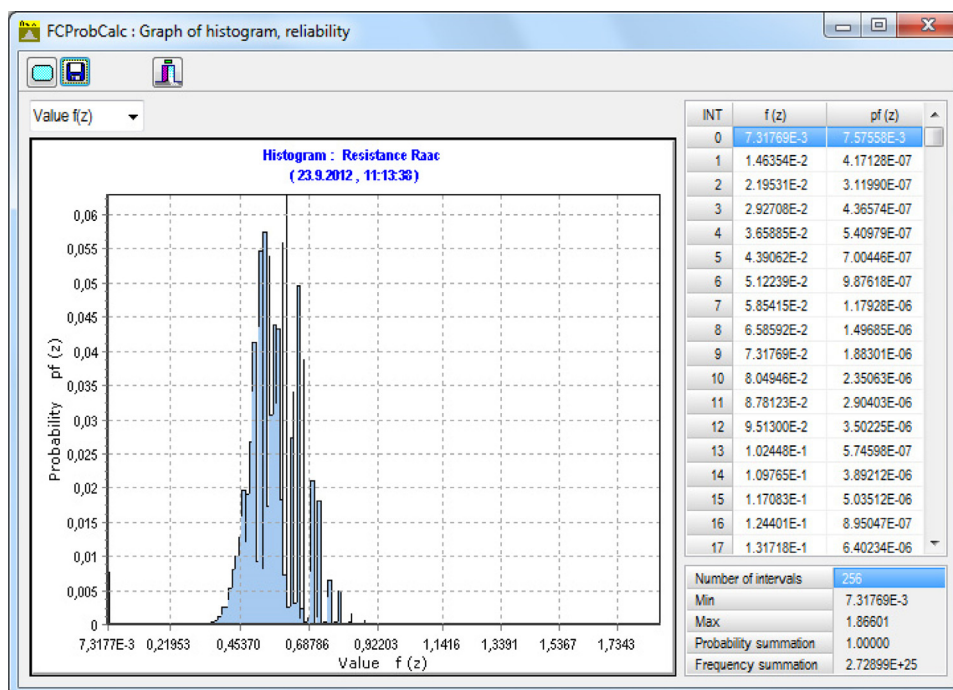
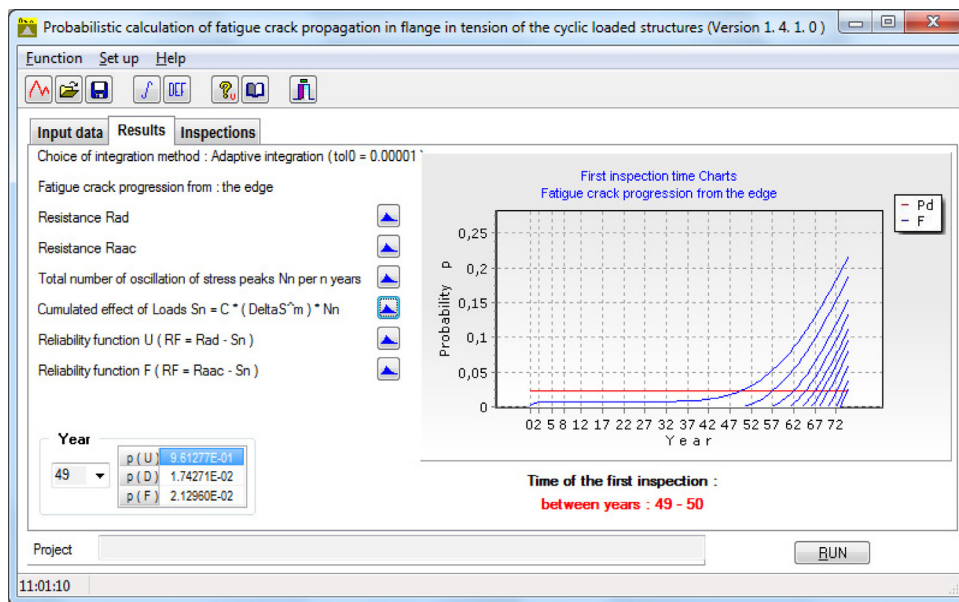


Figure 4 Resistance histogram –  $R_{(aac)}$  for the bridge structure subject to a fatigue crack from the edge (method: the adaptive method with numerical integration, the input parameter is  $tol_0 = 1 \times 10^{-4}$ ).



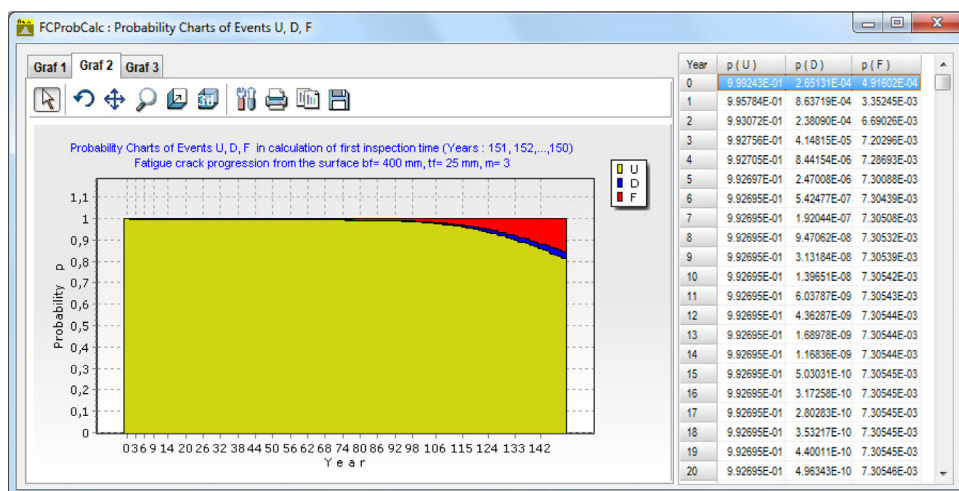
**Figure 5** FCProbCalc desktop with results of the probabilistic modelling of propagation of a fatigue crack from the edge. It was decided that the first inspection of the bridge should take place after 49 years of operation (method: the adaptive method with numerical integration, the input parameter is  $tol_0 = 1 \times 10^{-4}$ ).

of the bridge construction. When the probability of failure exceeds the specified designed probability,  $P_d$ , the inspection should be performed. It was decided that the first inspection of the bridge should take place after 49 years of operation. This inspection will focus on growth of the fatigue crack on the edge.

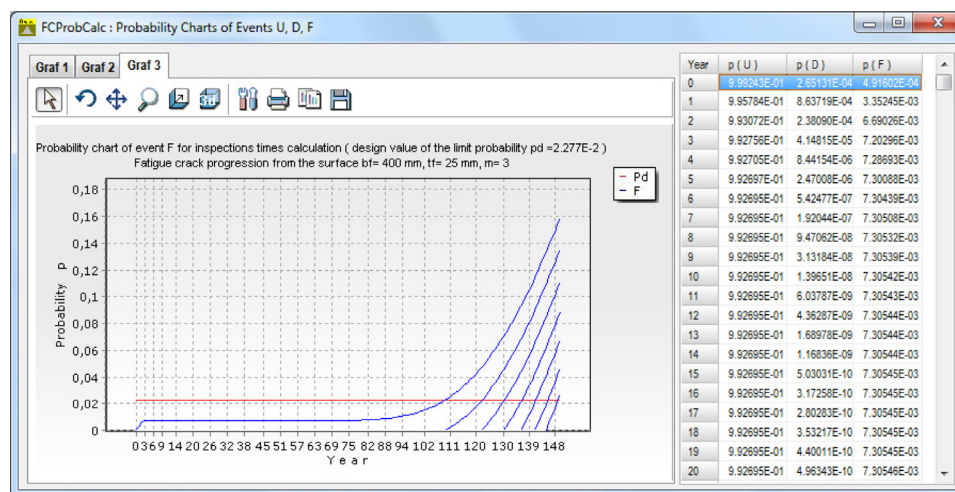
Similarly, it is possible to show the resulting probability of the elemental phenomena (11) through (13) for the probabilistic modelling of propagation of a fatigue crack from the surface – see Fig. 6, which proves validity of (14) because the sum of elemental phenomena (11) through (13)

equals to value 1. Fig. 7 shows the resulting times for the proposed inspections of the construction, a particular attention being paid to the growth of the fatigue crack from the surface.

It follows from the comparison of times for the first inspections which focus on the fatigue damage by the both types of the fatigue cracks (after 49 years of operation for the edge crack and after 109 years of operation for the surface crack) that the fatigue cracks propagate from the surface with a considerably lower speed that the fatigue cracks which initiate at the edge.



**Figure 6** The calculated probabilistic elemental phenomena,  $U$ ,  $D$  and  $F$ , in a bridge construction subject to cyclic loads from 0 to 150 years – the probabilistic modelling of propagation of a fatigue crack from the surface (Gaussian quadrature was chosen as a numerical integration method).



**Figure 7** Dependence of the failure probability,  $P_f$ , on years of operation of the bridge construction during the probabilistic calculation of propagation of fatigue cracks from the surface (0–150 years) with respect to the conditional probability and specification of the time for the first and subsequent inspections of the bridge structure (Gaussian quadrature was chosen as a numerical integration method).

## Conclusion

This paper discusses development of the DOProC probabilistic method and its use in the reliability assessment of the constructions. A particular attention is paid to the theory and practical aspects of the probabilistic assessment of the constructions which are subject to fatigue and tend to create fatigue cracks. The result of this method is similar to other probabilistic approaches: proposal of a system of regular inspections of the construction.

Those computations were applied in FCProbCalc which was used for the mathematical modelling of propagation of fatigue cracks from the edge and surface. A probabilistic reliability assessment of the constructions was also performed in this software – it was based on the exact definition of the permissible size of the fatigue crack. The probabilities were obtained for three basic phenomena which are related to propagation of the fatigue cracks. On the basis of those data, the probability of failure can be calculated for each year of operation of the construction. When determining the required degree of reliability, it is possible to specify the time of the first inspection of the construction which will focus on the fatigue damage. Using a conditional probability, times for subsequent inspections can be determined.

The methods and application can considerably improve estimation of maintenance costs for the structures and bridges subject to cyclical loads.

If this methodology is developed further, the goal of investigations seems to be, in particular, application of Bayesian networks in the computational model which describes propagation of fatigue cracks.

## Conflict of interest

The authors declare that there is no conflict of interest.

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## Appendix.

For a lite version of FCProbCalc and other software products based on DOProC visit <http://www.fast.vsb.cz/popv>.

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